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THE TRANSVERSE ELECTROMAGNETIC FIELD SUPPORTED BY AN INFINITELY CONDUCTING PLANE AND A PARALLEL INFINITELY CONDUCTING STRIP

(Unclassified)

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1. Introduction

In this report we address ourselves to the problem of determining the distribution of current in two perfect conductors, one an infinite plane and the other an infinitely long strip which is parallel to the plane. The geometry is shown in Figure 1. It is assumed that the strip

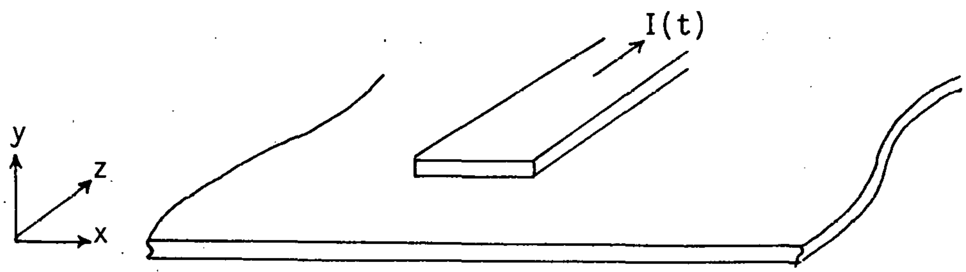

Figure 1
Current Carrying Strip Above
an Infinite Plane

carries a current $I(t)$ which returns via the plane.

It is well known that such a system can support a transverse electromagnetic, or TEM, wave. Therefore, using the TEM mode, we shall numerically calculate the electric and magnetic field in the region around the conductors and then apply boundary conditions to determine the surface current density on the conductors. By calculating the current density, we can then determine the degree to which the return current in the plane images the current in the strip.

2. The TEM Field

In this section we shall review the results concerning the propagation of TEM waves near a pair of perfectly conducting transmission line structures*. In particular, we restrict ourselves to the geometry depicted in Figure 1 where the region around the conductors is characterized by the constant parameters ε, μ, and σ. We describe the waves propagating along such a uniform system in terms of a propagation factor

$$e^{i\omega t-\gamma z} \tag{2.1}$$

with this factor in the electric and magnetic fields, Maxwell's equations in the dielectric region become

$$\begin{aligned}
&\text{(i)} && \frac{\partial E_z}{\partial y} + \gamma E_y = -i\omega\mu H_x \\
&\text{(ii)} && -\gamma E_x - \frac{\partial E_z}{\partial x} = -i\omega\mu H_y \\
&\text{(iii)} && \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -i\omega\mu H_z \\
&\text{(iv)} && \frac{\partial H_z}{\partial y} + \gamma H_y = i\omega\varepsilon E_x \\
&\text{(v)} && -\gamma H_x - \frac{\partial H_z}{\partial x} = i\omega\varepsilon E_y \\
&\text{(vi)} && \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\varepsilon E_z
\end{aligned} \tag{2.2}$$

where the components E_x, H_x, E_y, and so on, are functions of x and y only.

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* See Ramo, Whinnery, and Van Duzer [1] for a detailed discussion

Waves which contain neither electric or magnetic fields in the direction of propagation, i.e.

$$E_z = H_z = 0\,, \tag{2.3}$$

are called transverse electromagnetic (TEM) waves. From equations (2.2) we obtain, using (2.3), the following equations for TEM waves:

$$\begin{aligned}
&\text{(i)} && \gamma E_y = -i\omega\mu H_x \\
&\text{(ii)} && \gamma H_y = i\omega\varepsilon E_x \\
&\text{(iii)} && \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \\
&\text{(iv)} && \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0
\end{aligned} \tag{2.4}$$

By equating (2.2i) and (2.2v), and using (2.3), we also obtain the condition

$$\gamma^2 + \omega^2\varepsilon\mu = 0$$

or

$$\gamma = \pm i\omega\sqrt{\varepsilon\mu} \tag{2.5}$$

for the progation constant γ. Finally, using (2.3) along with the fact that

$$\operatorname{div}\vec{E} = 0 \qquad \operatorname{div}\vec{H} = 0$$

we obtain for waves containing the factor (2.1) the equations

$$\begin{aligned}
&\text{(i)} && \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = 0 \\
&\text{(ii)} && \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} = 0
\end{aligned} \tag{2.6}$$

Therefore, we see that the transverse electric field and the transverse magnetic field in the transverse coordinates satisfy the same equations as in the electrostatic and magnetostatic case. From equation (2.4 iii) we conclude that there exists a potential function $u = u(x,y)$ for which

$$\vec{E} = -\text{grad } u \tag{2.7}$$

This last statement follows from Green's Theorem and the fact that the line integral $\int_C E_x dx + E_y dy$ vanishes for all closed curves C in the transverse plane.

Consequently, the problem of determining the surface current density $\vec{J}_s$ can be reduced to the problem of finding the potential function u. The procedure will be to calculate u which satisfies Laplace's equation

$$\text{div grad } u = 0 , \tag{2.8}$$

Then, equation (2.7) can be used to calculate the transverse electric field; subsequently, equations (2.4i) and (2.4ii) will give the components of the transverse magnetic field $\vec{H}$. Finally, on the boundary of the conductors we must satisfy the condition

$$n \times \vec{H} = \vec{J}_s \tag{2.9}$$

which yields the surface current density. The electric field is of course normal to the surfaces of the conductors.

3. Solution for the Potential

Solutions to (2.8) for the potential u will be approximated using the method of successive relaxation, an iterative finite difference scheme (see Isaacson and Keller [2] or Gary [3] , We begin by setting up the boundary value problem for u . Figure 2 shows a cross-section of the two-conductor system displayed

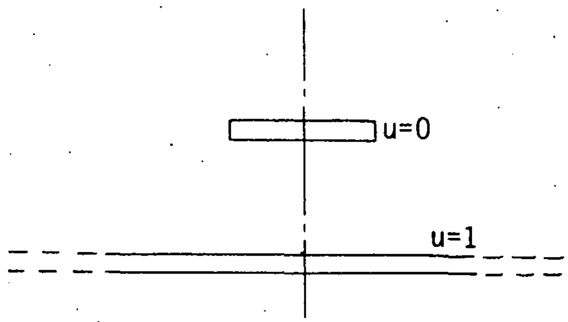

Figure 2
Normalized Boundary Conditions for u
in Transverse Plane

in Figure 1 where we have set $u = 0$ on the boundary of the lower conducting plane and $u = 1$ on the boundary of the conducting strip. Taking advantage of the symmetry and introducing "boundaries at infinity", we obtain the boundary value problem indicated schematically in Figure 3, where $\partial u/\partial n$

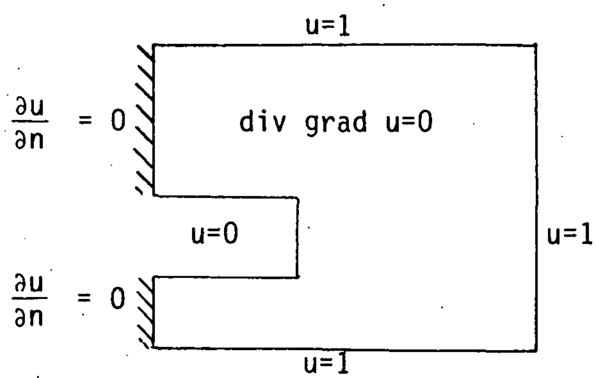

Figure 3

Boundary Value Problem for the
Potential u

denotes the normal derivative $\partial u/\partial n = \vec{n} \cdot \text{grad } u$.

As aforementioned, the boundary value problem shown in Figure 3 can be solved numerically by finite difference methods. The Appendix contains the details of this calculation as well as the FORTRAN program. In the present section, we shall discuss the numerical results of this computation in two special cases. These results were obtained by running the code on a CDC 7600 at Lawrence Livermore Laboratory.

In the first case we considered a multiconductor system in which the strip was .003 inches thick, .500 inches wide, and was situated .020 inches above the plane. Secondly, we considered a strip .0005 inches thick, .05 inches wide, and situated .010 inches above the ground plane. We shall refer to these as System I and System II, respectively. The code was then run for each system and the magnitude of the electric field (which is proportional to the magnitude of the magnetic field and hence proportional to the surface current density) was calculated on the upper and lower surface of the strip and on the surface of the ground plane. Figures 4 and 5 give the results of the calculations for the two systems. We can observe from Figures 4 and 5 that in the case of perfect conductors, there is quite a bit of current imaging in the ground plane.

By running the code for several different geometries, we can make the following general conclusions concerning the distance of separation, thickness of the strip, and imaging in the ground plane. First, we remark that the current through the strip is given by

$$I = \oint_{\substack{\text{bdry of} \\ \text{strip}}} (\vec{H} \times \vec{n})\, d\ell \tag{3.1}$$

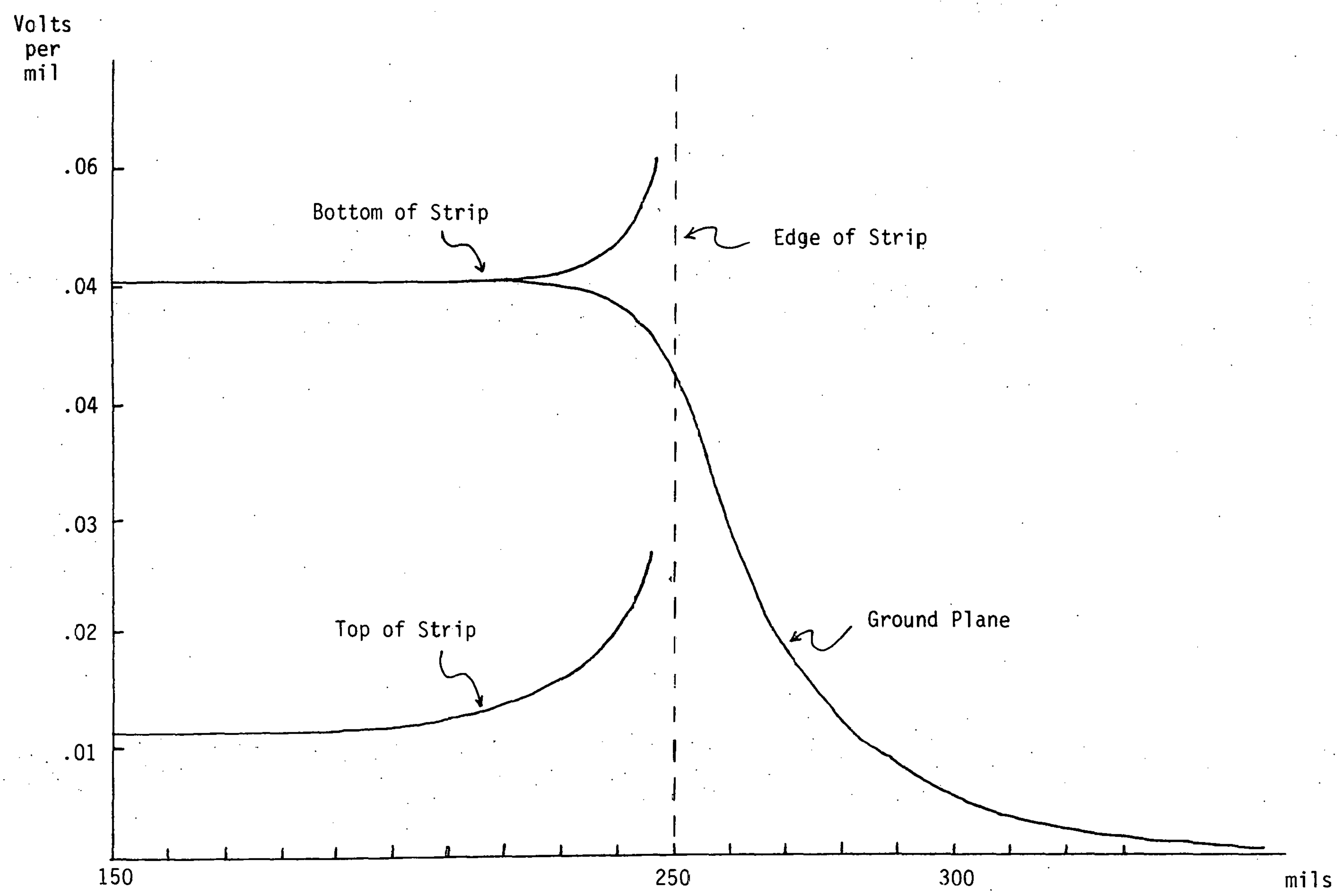
Volts per mil
.06
.04
.04
.03
.02
.01
Bottom of Strip
Edge of Strip
Top of Strip
Ground Plane
150
250
300
mils

Figure 4

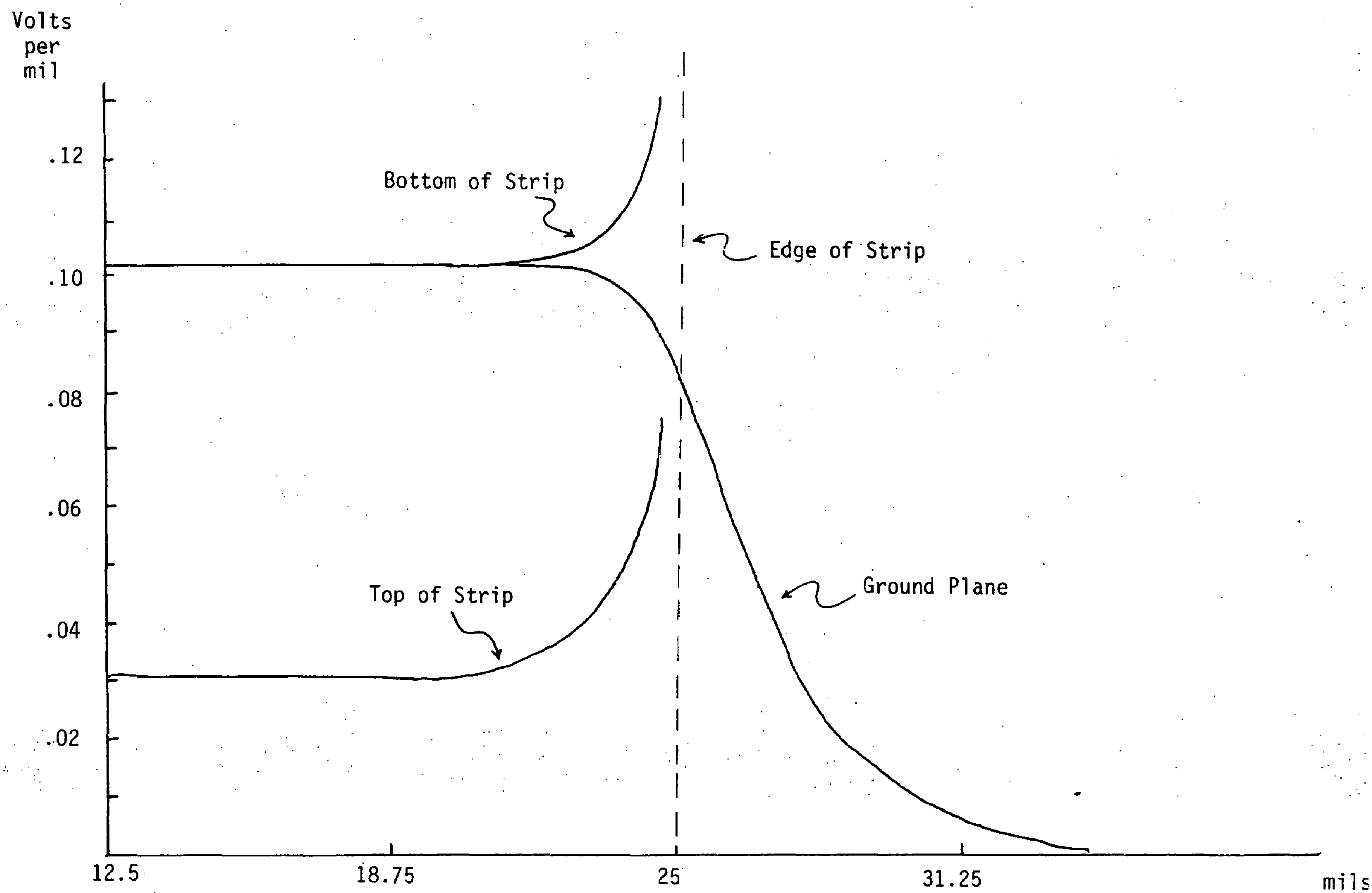
Volts per mil
.12
.10
.08
.06
.04
.02
Bottom of Strip
Edge of Strip
Top of Strip
Ground Plane
12.5
18.75
25
31.25
mils

Figure 5

For the same value of the current and the same thickness of strip, increasing the distance between the strip and the plane tends to decrease the electric field in the region between the strip and plane and also there is less imaging in the ground plane. Moreover, for a given current and fixed distances above the ground plane, increasing the thickness of the strip will, to a slight degree, weaken the field in the region between the conductors and cause less imaging. The reason for this small effect is that most of the current will lie on the lower side of the conductor, as Figures 4 and 5 show.

Finally, for only passing interest, a typical field is sketched in Figure 6.

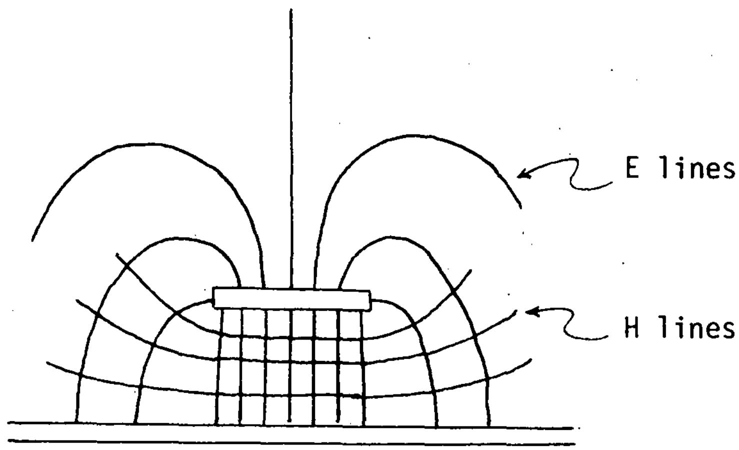

Figure 6
TEM fields in
Two-Conductor System

Acknowledgment

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References

[1] S. Ramo, J. R. Whinnery, T. Van Duzer, "Fields and Waves in Communication Electronics", John Wiley and Sons, Inc., New York (1965).

[2] E. Isaacson and H. B. Keller, "Analysis of Numerical Methods", John Wiley and Sons, New York (1966).

[3] J. Gary, "The Numerical Solution of Partial Differential Equations", NCAR Manuscript No. 69-54, Boulder, Colo. (1969).

APPENDIX

The numerical code, which is written in FORTRAN IV, solves the boundary value problem shown in Figure 3. Basically, the code zones the region under consideration, initializes the potential array U(J.K), calculates the potential by successive relaxation, and finally calculates the normal derivatives of u at the boundaries of the conductors. In a NAMELIST the user is required to input the dimensions L1, L2, L3, L4, and L5 (see Figure 7), the numbers M and N of horizontal and vertical zones, and the number of iterations ITER. Figure 7 also defines the zoning parameters.

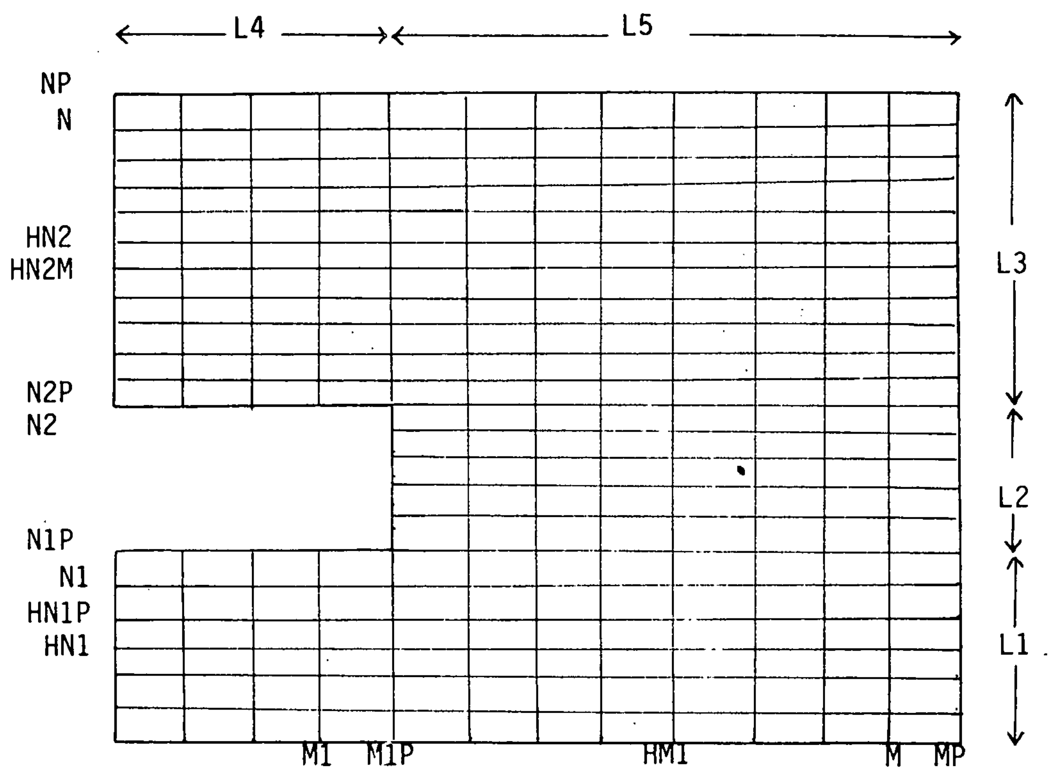

Figure 7
Dimension and Zoning
Parameters

List of FORTRAN Variables

L1,L2,L3,L4,L5	Dimension of the region (Fig. 7)
ITER	number of iterations in successive relaxation
M	number of horizontal zones
N	number of vertical zones
DX	Δx, horizontal zone dimension
DY	Δy, vertical zone dimension
N,NP,HN2,HN2M,N2, N2P,N2P2,N1,N1P,HN1, HN1P,M1,M1P,HM1,M,MP	Variables related to zoning (Fig. 7)
U(J,K)	an array for the potential function at each grid point (J,K)
G(I)	temporary array to store derivatives at the boundary of the conductor
R	Δx^2
S	Δy^2
T	$^1/(\Delta x^2 + \Delta y^2)$

```
C  THIS CODE COMPUTES THE POTENTIAL FUNCTION IN A REGION
C  AROUND TWO PERFECT CONDUCTORS WHICH ARE SUPPORTING A
C  TRANSVERSE ELECTROMAGNETIC FIELD.  THE USER MUST SUPPLY
C  THE DIMENSIONS OF THE REGION L1,L2,L3,L4,L5, THE NUMBER
C  OF HORIZONTAL AND VERTICAL ZONES M AND N, AND THE NUMBER
C  ITER OF ITERATIONS USED IN SUCCESSIVE RELAXATION. THESE
C  DATA ARE CONTAINED IN NAMELIST NAM1.  SETTING ISTOP EQUAL
C  TO UNITY STOPS THE PROGRAM.
      PROGRAM TEM(TAPE59)
      REAL L1,L2,L3,L4,L5
      INTEGER HN1,HN2,HM1,HM1M,HN2M,HN1P
      DIMENSION U(25,100),G(25)
      DATA ISTOP,ITER/0,250/
      NAMELIST/NAM1/L1,L2,L3,L4,L5,M,N,ITER,ISTOP
  121 READ(59,NAM1)
      IF(ISTOP.EQ.1) GO TO 120
      WRITE(59,100) L1,L2,L3,L4,L5,M,N,ITER
  100 FORMAT(* INPUT = *,5F7.2,3I6,//)
      DX=(L4+L5)/M
      DY=(L1+L2+L3)/N
C  THE FOLLOWING PARAMETERS ARE RELATED TO ZONING
      N1=L1/DY
      N1P=N1+1
      N1P2=N1+2
      N2=(L1+L2)/DY
      N2P=N2+1
      N2P2=N2+2
      M1=L4/DX
      M1P=M1+1
      M1P2=M1+2
      MP=M+1
      NP=N+1
      R=DX**2
      S=DY**2
      T=1.0/(R+S)
      HM1=M1+(M-M1)/2
      HM1M=HM1-1
      HN2=N2+(N-N2)/2
      HN2M=HN2-1
      HN1=N1/2
      HN1P=HN1+1
CTHE FOLLOWING INITIALIZES THE ARRAY U(J,K)
      DO 21 J=1,MP
      DO 21 K=1,HN1
   21 U(J,K)=0.
      DO 22 J=HM1,MP
      DO 22 K=HN1P,NP
   22 U(J,K)=0.
      DO 23 J=1,HM1M
      DO 23 K=HN2,NP
   23 U(J,K)=0.
      DO 24 J=1,HM1M
      DO 24 K=HN1P,N1P
   24 U(J,K)=1.
```

```
      DO 25 J=M1P,HM1M
      DO 25 K=N1P2,N2
25    U(J,K)=1.
      DO 26 J=1,HM1M
      DO 26 K=N2P,HN2M
26    U(J,K)=1.
      DO 27 J=1,M1
      DO 27 K=N1P2,N2
27    U(J,K)=7.0
C  USE SUCCESSIVE RELAXATION TO APPROXIMATE THE POTENTIAL
C  FUNCTION U(J,K) IN THE REGION
      DO 200 L=1,ITER
      DO 11 K=2,N1
      U(1,K)=T*(R/2.*(U(1,K-1)+U(1,K+1))+S*U(2,K))
      DO 11 J=2,M
11    U(J,K)=T/2.*(R*(U(J,K-1)+U(J,K+1))+S*(U(J-1,K)+U(J+1,K)))
      DO 12 K=N1P,N2P
      DO 12 J=M1P2,M
12    U(J,K)=T/2.*(R*(U(J,K-1)+U(J,K+1))+S*(U(J-1,K)+U(J+1,K)))
      DO 13 K=N2P2,N
      U(1,K)=T*(R/2.*(U(1,K-1)+U(1,K+1))+S*U(2,K))
      DO 13 J=2,M
13    U(J,K)=T/2.*(R*(U(J,K-1)+U(J,K+1))+S*(U(J-1,K)+U(J+1,K)))
200   CONTINUE
      GO TO 77
      DO 99 I=1,NP
      K=NP-I+1
99    WRITE(59,201) (U(J,K),J=1,MP)
77    CONTINUE
201   FORMAT(15F5.2)
C  CALCULATION OF THE GRADIENT AROUND THE BOUNDARY OF THE
C  UPPER CONDUCTOR
      WRITE(59,300)
300   FORMAT(//,* GRAD AROUND THE UPPER CONDUCTOR*,//)
C     THE UPPER BOUNDARY
      DO 901 I=1,M1
901   G(I)=(U(I,N2P2)-1.0)/DY
      WRITE(59,301)(G(I),I=1,M1)
301   FORMAT(10F6.4)
C     THE RIGHTMOST BOUNDARY
      DO 902 I=N1P2,N2
      G(I)=(U(M1P2,I)-1.0)/DX
902   WRITE(59,302) G(I)
302   FORMAT(35X,F6.4)
C     THE LOWER BOUNDARY
      DO 903 I=1,M1
903   G(I)=(1.0-U(I,N1))/DY
      WRITE(59,301)(G(I),I=1,M1)
      WRITE(59,303)
303   FORMAT(//,*GRAD ON THE LOWER CONDUCTOR*,//)
      DO 904 I=1,MP
904   G(I)=U(I,2)/DY
      WRITE(59,301)(G(I),I=1,MP)
      GO TO 121
120   CALL EXIT
      END
```

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